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HOMOGENIZATION OF SECOND ORDER EQUATION WITH SPATIAL DEPENDENT COEFFICIENT

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Dedicated to Professor Hwai-Chiuan Wang on his retirement.

Abstract. We establish the homogenization of the boundary value problem of a second order differential equation. It generates nonlocal effect. The eigenfunction expansion and Fredholm integral equation are exploited to obtain a characterization of the kernel while in the space independent case the Young measure is applied to obtain the explicit formula of the kernel.

1. Introduction We study the behavior as ϵ goes to zero of solutions u^ϵ of the second order differential equation

$$\begin{aligned} \mathcal{L}^\epsilon u^\epsilon(x,t) \equiv -\partial_x^2 u^\epsilon(x,t) + a^\epsilon(x,t)u^\epsilon(x,t) = \lambda u^\epsilon(x,t) + f(x,t)\,, \\ u^\epsilon(0,\cdot) = u^\epsilon(\pi,\cdot) = 0\,, \end{aligned} \tag{1.1}$$

where $f(x,t), a(x,t) \in C([0,\pi]\times[0,T])$, $(x,t)\in[0,\pi]\times[0,T]$. Here λ is a parameter, while the sequence of measurable functions defined by $a^\epsilon(x,t) = a(x,\frac{t}{\epsilon})$ satisfies the bounds

$$\alpha \le a^\epsilon(x,t) \le \beta\,, \qquad \text{a.e. in } [0,\pi]\times[0,T]\,, \tag{1.2}$$

and is equicontinuous in x, i.e., there is a function φ such that $\varphi(\sigma)\to 0$ as $\sigma\to 0$ and

$$|a^\epsilon(x,t) - a^\epsilon(z,t)| \le \varphi(|x-z|)\,. \tag{1.3}$$

The theory of homogenization is then concerned with understanding how oscillations of coefficients $\{a^\epsilon(x,t)\}_\epsilon$ of (1.1) create oscillations in its solutions. We will show that the limit operator $\mathcal{L}^0$ of the differential operators $\{\mathcal{L}^\epsilon\}_\epsilon$ is an integro-differential operator. This limiting operator is showing the *nonlocal effect*.

The nonlocal effects which may appear by homogenization had first been noticed by Enrique Sanchez-Palencia [20] (using asymptotic expansions in a periodic setting), for questions like Visco-Elasticity or for some memory effects in Electricity

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